Lemons are Green: The Informative Role of a Pigovian Tax

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March 13, 2006

Abstract

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When households are not fully informed about the environmental concern of a new policymaker, the latter may use environmental taxes as signals for environmental friendliness. In this context, the "lemons" are the green policymakers: they have an incentive to distort upward environmental taxes in an attempt to masquerade as brown policymakers. To fully reveal her environmental concern, the brown policymaker must pay a signaling cost by overtaxing, hence polluting less than required by the Pigovian principle. Selection among perfect Bayesian Equilibria shows that uninformative taxes may prevail over informative taxes depending on households' prior beliefs about the policymaker's preferences.

Keywords: Pigovian tax, Signaling.

JEL Code: D82, H21, H23, Q28.

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1 Introduction

As pointed by the large and rapidly growing literature on optimal taxation (see, for instance, Bovenberg and van der Ploeg (1994) or Bovenberg and de Mooij (1994)), the environmental tax is widely regarded as both an efficient instrument for internalizing the marginal social damage from pollution and an effective device for raising public revenue. The usual assumption made by the literature that information is perfect casually dismisses the question of how private information interacts with policymakers' behaviors.

When a new policymaker comes into office, households might not be fully informed about her concern for the environment. However, households can make inferences about the environmental friendliness of the policymaker by observing policy actions such as the choice of an environmental tax that is made by the policymaker. In this paper, we present a model based on such an asymmetric information, in which environmental taxes may potentially be used as signals for the unobservable policymaker's concern for the environment. Identifying this concern with the environmental damage resulting from the policymaker's behavior, we investigate conditions under which an environmental tax is fully revealing the severity of the damage, hence the policymaker's concern for the environment. It turns out that households' prior beliefs about the policymaker's concern for the environment matter in selecting among multiple equilibria, which are as usual of two kinds: either separating equilibria in which the policymaker transmit all information on her environmental concern by committing on the environmental tax early in her term of office, or pooling equilibria in which "green" and "brown" policymakers choose tax alike and so convey no information to households on their true concern for the environment.

Perhaps the most surprising result of the model is the emergence of opportunistic behaviors coming from green policymakers, due to their informational advantage. When households believe more likely a policymaker to be brown at the time she comes into office, the latter has no incentive to thwart this belief, even though she is actually green. The green policymaker may rather raise the environmental tax above the Pigovian tax, which fully internalizes the marginal social damage from pollution, in an attempt to imitate the behavior of a brown policymaker. This increases the households' perceived utility, thereby boosting the private component of social welfare. Obviously, pooling strategies entail a welfare loss to the brown policymaker since the environmental tax falls short of the actual marginal production damage from pollution caused by her policy. This loss, however, may be less than the cost that the brown policymaker should incur to signal her environmental concern to households, especially when they initially believe more likely that the policymaker is brown. As a result, separation is achieved with perfectly informative taxes when consumers hold high prior beliefs that the policymaker is green, whereas pooling equilibrium taxes emerge when consumers initially assign a high probability to the brown policymaker. In some sense, pessimistic beliefs on the policymaker's concern for the environment freeze information.

There is some analogy with the lemons problem described by Akerlof (1970): "bad cars sell at the same price as good cars since it is impossible for a buyer to tell the difference between a good and a bad car; only the seller knows". Similarly, a green policymaker sets the same environmental tax as a brown policymaker because households cannot ascertain the environmental friendliness of the policymaker and, moreover, hold pessimistic prior beliefs about her environmental concern. Here, however, the bad type who has an incentive to withhold the truth is the green policymaker with a high-quality environment. As far as the environmental friendliness of policymakers is concerned, lemons are green.

There are a variety of reasons why households and policymakers are likely to have different informations and beliefs about the degradation of the natural environment, hence different preferences concerning the policy stance on the environment. A first reason given by Boyer and Laffont (1999) is that the policymaker generally has superior knowledge of the environmental damage because she is endowed with superior data from confidential reports of the public service bureaucracy. A second reason also mentioned by Boyer and Laffont (1999) is that the policymaker's scientific information on nonverifiable environmental variables may be costly to communicate. This argument is reinforced by the evidence that scientific opinion on the extent to which pollution harms the environment is changing rapidly as new information comes to light. Recalling the experience of the Montreal Protocol, Pearce (1991) notes that "the scale of the threat from chlorofluorocarbons has been revised upwards several times". A third reason households have perceptions of the environmental damage different from the policymaker's perceptions is because the mass media systematically confront the public with a lopsided version of reality giving the impression that the global environment is in the worst shape[1]. Even if households were little influenced by the images and messages on television and in the newspapers, the assumption that they have the same information about the environmental damage as does the policymaker is unlikely to be met in reality because households usually lack the technical ability required for a sound assessment of the environmental damages.

The consequences of asymmetric information between households and the

[1]According to Lomborg (2001), the environmental message delivered on television and in the newspapers is generally characterized by a tendency to overemphasize pessimistic viewpoints. To illustrate what he calls the "Litany" of our ever deteriorating environment, Lomborg quotes assertions found in the newspapers such as "everyone knows the planet is in bad shape" (Herstgaard (2000), "A global green deal", Time 155 (17): 84-85), or "the burning of fossil fuels is endangering the lives of millions... we are heading for cataclysm" (The April 2001 Global Environment Supplement from *New Scientist*).

policymaker in the present context is that consumption decisions regarding polluting goods may deviate from what would be socially optimal from the policymaker's viewpoint. To the extent that the perceptions by households of the environmental concern of the policymaker are influenced by her behavior, the choice of environmental taxes by the policymaker has a role to play in eliminating the difference in perceptions. This extends the role of the environmental taxes beyond that, traditionally emphasized by the environmental literature, of an efficient instrument for internalizing the adverse external effects of pollution. However, using environmental taxes to convey information to households may in turn lead the policymaker to distort such taxes from levels implied by the Pigovian principle.

In a first version of the model, the policymaker cannot commit herself to a level of environmental tax. Hence, the beliefs held by households about the preferences of the policymaker and the corresponding environmental damage are not influenced by the policymaker's decisions. In equilibrium, the policymaker's choice of the so-called "discretionary" tax is not only a weighted average of the polluting externalities caused by each household's consumption as in Diamond (1973), but it also takes into account the difference in beliefs between the policymaker and the households. If households perceive the environmental damage to be higher (resp. lower) than what the policymaker actually thinks, then the policymaker must distort the tax downward (resp. upward) relative to what would require the Pigovian principle were households to have the same beliefs as the policymaker. The analysis of the discretionary tax policy provides the preliminary and crucial insight that the policymaker prefers households to perceive herself as brown, regardless of her true concern for the environment. When households believe more likely that the policymaker is little concerned about the environment, they expect the polluting good to be cheap since cleanup costs are thought to be low. Hence, pessimistic beliefs boost the private component of social welfare.

The second version of the model assumes that environmental taxes are enforceable. The policymaker's commitment to abide by her tax choice may come from the rule of law or the policymaker's willingness to build up a reputation for the credibility of her environmental policy. If households are not fully informed about the policymaker's concern for the environment in a regime that involves some commitment on tax, they will condition their beliefs on environmental taxes and the policymaker will choose how much information to reveal by setting taxes. We investigate the existence and the nature of perfect Bayesian equilibria in this signaling context. It is shown that separating equilibria exist in which the policymaker transmits all information on her environmental concern with a tax that is above (resp. equal to) the symmetric information Pigovian level when she is brown (resp. green). This case arises when the welfare obtained by the brown policymaker with her symmetric information Pigovian tax makes the green policymaker envious. Employing the logic of *Undefeated Equilibrium* proposed by Mailath, Okuno-Fujiwara and Postlewaite (1993), we single out the separating equilibrium in which a brown policymaker distorts the environmental tax the minimum necessary to persuade households that she is actually brown, and a green policymaker behaves according to the Pigovian principle under symmetric information. A rough intuitive explanation for this surprising result is easy to provide. There are two components of social welfare, namely private welfare which depends on households' subjective beliefs, and environmental quality. Private welfare is affected by how much information is transmitted. Households have the impression to get higher utility when they believe the policymaker to be brown, because the dirty good is expected to be cheaper in a more polluted environment. Thus, a green policymaker has an incentive to mimic the brown policymaker and impose a high tax in order to trick households into believing that the environmental damage is severe. Under precise circumstances, the only way for a brown policymaker to transmit all information on her type is to distort the environmental tax, hence reduce polluting wastes, even more than what would be implied by the Pigovian principle under symmetric information. Necessary conditions for environmental taxes to be informative are that, first, increasing tax is more detrimental to the green policymaker than it is to her brown counterpart, and second, cleanup costs are lower when the policymaker is brown. Under such conditions, upward deviations from the brown policymaker's Pigovian tax emerge when the welfare obtained with this tax makes the green policymaker envious.

The selection of *undefeated equilibria* shows, however, that pooling equilibria prevail over the least-cost separating equilibrium when households believe more likely the policymaker to be brown. In other words, no information will be revealed through environmental taxes when households have pessimistic prior beliefs about the policymaker's concern for the environment. In such circumstances, the green policymaker has a strong incentive to mimic, which raises the signaling cost for the brown policymaker. The latter is better off concealing her true environmental concern from households even though the undefeated pooling equilibrium taxes are distorted downward relative to the symmetric information Pigovian level, which yields more polluting wastes and a welfare loss. By contrast, undefeated pooling equilibrium taxes reduce pollution even more than under the symmetric information Pigovian level since they are distorted upward.

Our conclusion that environmental taxes may be distorted for reasons of signaling contributes to the existing literature in identifying the various causes of potential deviations from the Pigovian tax. Considering monopolistic competition, Barnett (1980) has clarified Buchanan's observation in Buchanan (1969) that underproduction of a monopoly and the overproduction due to an externality may compensate each other so that second best optimal taxes may be less than marginal environmental damages. Besides market power, the assumption of asymmetric information, especially between the environmental regulator and economic agents, has provided noteworthy reasons why optimal environmental taxes should deviate from levels implied by the Pigovian principle. The context of asymmetric information that has received greatest attention is that pioneered by Baron (1985) and accurately investigated thereafter by Laffont (1994) and Lewis (1996): polluters are more informed than regulators about the costs and benefits of pollution reduction. A general result emerging from this literature is that environmental taxes should be used by the regulator as means of providing polluters with the correct incentives for fully reporting their information. Along this line, Laffont (1994) examines to what extent Pigovian taxes are distorted so as to reduce the polluter's rent of asymmetric information.

Our model essentially differs from the incentive regulation literature in that the policymaker has an informational advantage over economic agents. Perhaps the closest relationship is with the hypothesis in Barigozzi and Villeneuve (2004) that agents are less informed than the government on the effect of their consumption. The authors analyze optimal taxation in a second-best world in which raising public funds is costly for the government. As a result, the latter must distort the Ramsey-Sandmo tax downward to fully reveal to agents the detrimental effect of their consumption. The presence of social costs of raising public funds proves to be crucial for the emergence of signaling distortions. By contrast, no such costs are assumed in our framework which deliberately deals with a world without distortionary taxes when symmetric information prevails. A second major difference is that Barigozzi and Villeneuve consider that the government is privately informed on the adverse effects of individual consumption. In this paper, the policymaker is privately informed on both her environmental preferences and the polluting externalities due to households' consumption.

Other authors have explored the strategic behavior of a policymaker under the assumption that households are uncertain about her preferences. For instance, Barro (1986) builds a multiperiod model on such an assumption to show that a monetary authority may find it optimal to commit herself to a rule and build up a reputation that will cause households to believe her announcements. The presence of asymmetric information induces a policymaker to keep initial inflation down for reasons of signaling in the two-period analysis of Vickers (1986)). When voters are not fully informed about the preferences of the policymaker, the electoral behavior of the latter is shown by Alesina and Cukierman (1990) to be more ambiguous than under complete information. All these works, like ours, draw on the seminal articles in the literature of industrial organization by Milgrom and Roberts (1982 and 1986) who considered the first models in which a firm uses prices, among other variables, to signal its private information either on production costs or on product quality.

The signaling game investigated here shares some essential structural properties with the signaling games that have been extensively analyzed in Cho and Kreps (1987) and Cho and Sobel (1990) on the basis of the wellknown model of Spence (1974). First, the types of the policymaker can be ordered in such a way that the brown policymaker is more willing to choose higher taxes than is the green policymaker. This is another instance of the familiar single-crossing property. Another usual property which is satisfied by our model is stochastic dominance: whatever her true environmental concern, the policymaker prefers households to believe her more likely to be brown.

Moreover, the present model is somewhat reminiscent of the informed principal's model with common values developed by Maskin and Tirole (1992) in that the private information of the policymaker who plays here the role of a principal, is an argument of the household-agent's objective function. A major difference is that, in our setting, the signaling activity takes place before contracting in the spirit of the Spencian education model. A common feature with Maskin and Tirole (1992) is that the concept of *Weakly Interim Efficiency* introduced by these authors to restrict the set of incentive compatible allocations has the same flavor as the logic of *Undefeated Equilibrium* proposed by Mailath, Okuno-Fujiwara and Postlewaite (1993), which is used here to refine equilibria and focus on the most "reasonable" ones.

The paper is organized as follows. Section 2 presents a model of environmental taxation under asymmetric information and states some useful properties. Section 3 investigates discretionary taxation by assuming that the policymaker cannot make binding commitments. Section 4 analyzes the enforceable tax policy and the role of environmental tax as a signal of the policymaker's environmental concern. Section 5 offers conclusions and proposes some extensions of the model.

2 The model

A new policymaker has come into office in an economy made up by N households with identical preferences. The households cannot ascertain the environmental friendliness of the policymaker. They believe there are two possible types of policymaker, the brown and the green. The brown policymaker cares less about the environment than the green, hence the brown policy is characterized by laxer cleanup standards and more severe damages from pollution than is the green policy. A coefficient of polluting wastes denoted by $\widehat{\varepsilon}$ reflects the policymaker's concern for the environment. This coefficient is not observed by the households *ex ante*. It can be considered as an estimate of the environmental damage on which the policymaker in office has superior information because she is better equipped than households, as suggested by Boyer and Laffont (1999). The information initially available to households is described as follows: they believe the policymaker is brown (ε^b) with a commonly known probability $\mu_0 \in (0, 1)$, and green (ε^g) with probability

$1-\mu_0$, and $0<\varepsilon^g<\varepsilon^b$[2]. Let $\widehat{\varepsilon}(\mu)\equiv\mu\varepsilon^b+(1-\mu)\,\varepsilon^g$ be the households' perception of the environmental damage when they assign probability μ to the brown policymaker. The household's utility depends on one public good, namely environmental quality E, and two private goods: consumption of a polluting good x and labor l which is taken as the numeraire. Assume that environmental quality does not directly affect private demand and there are no income effects on the polluting good sector, hence the representative household i's utility function $U(x_i,l_i,E)$ takes the quasi-linear form:

$$U(x_i,l_i,E)=u(x_i)+E(\frac{\Sigma_{i=1}^N x_i}{N},\widehat{\varepsilon}(\mu))-l_i, \tag{1}$$

where $u(x_i)$ is bounded above and twice differentiable with $u''(x_i)<0$[3] at all $x_i\geq 0$ and $u(0)=0$. Throughout the paper, subscripts i (resp. superscripts j) and masculine (resp. feminine) pronouns refer to households (resp. policymaker).

Environmental quality deteriorates with pollution, which is directly related to the average consumption of the polluting good $\overline{x}=\frac{\Sigma_{i=1}^N x_i}{N}$ according to the linear relationship $E(\overline{x},\widehat{\varepsilon}(\mu))=-\widehat{\varepsilon}(\mu)\overline{x}$: this corresponds to measuring wastes in units proportional to the social damage caused by pollution.

[2] The assumption that households' information is binary, i. e., valuation $\widehat{\varepsilon}$ lies in the finite set $\{\varepsilon^g,\varepsilon^b\}$, is without loss of generality in the sense that it could easily be extended to the case of a random variable described by a cumulative distribution function and a density whose support is some interval $\left[\varepsilon^g,\varepsilon^b\right]$.

[3] As usual, primes denote derivatives.

With subjective belief μ, $-\widehat{\varepsilon}(\mu)\overline{x}$ represents the households' perception of the environmental damage (or environmental benefit from pollution reduction).

The polluting good is produced from labor alone with a constant-returns-to-scale technology. Consumption of the good is assumed to leave a waste residue. This requires cleanup activities that consume economic resources. The whole technology (production plus cleanup) is represented by the following cost function $c(x, \varepsilon) = c(\varepsilon)x$ which incorporates the cleanup costs. Cleanup costs depend on the environmental target of the policymaker. Consider for instance a junk automobile. With a brown policymaker, the junk automobile will be abandoned on the parkway or the river bank, whereas a green policymaker will charge car manufacturers the incremental costs of collection and disposal. As the preferences of the green policymaker for a low environmental damage involve high costs of cleanup for firms, a natural assumption is $c'(\varepsilon) < 0$[4], hence marginal production costs are higher with a green type than with a brown type. Firms are assumed to have the same information about the policymaker's concern for the environment as do households. When the public believes that the policymaker is the brown type with probability μ, the marginal cost of producing the polluting good is expected to be $\widehat{c}(\mu) \equiv \mu c(\varepsilon^b) + (1-\mu)c(\varepsilon^g)$.

At the time of trading the polluting good, firms facing uncertain cleanup

[4]The idea that a cleaner environment requires adjustment in cleanup at the expense of more labor input is similar to that in Copeland and Taylor (1994) or Yu (2005) for an abatement technology.

costs make the production decision that determines the price p of the polluting good. Competition among firms drives profits to zero, thus equilibrium is achieved in the polluting good market when $p = \widehat{c}(\mu)$[5]. If the true production costs $c\left(\varepsilon^{j}\right)$ turn out to be lower (higher) than the expected cost $\widehat{c}(\mu)$, then profits will be positive (negative). Let $\Pi^{j}\left(\mu\right) \equiv \left[\widehat{c}(\mu) - c\left(\varepsilon^{j}\right)\right] \Sigma_{i=1}^{N} x_i$ denote the aggregate expected profits.

The policymaker can levy a green tax t on polluting consumption. Let $\mathcal{T} \equiv [0, +\infty)$ be the set of possible taxes for each policymaker. It will be assumed, as in Diamond (1973), that any tax revenue is returned to households via lump-sum transfers T.

In addition to his wage income and the lump-sum transfer T/N, each household receives income from a share s of the aggregate expected profits $\Pi^{j}\left(\mu\right)$, which is no higher than $1/N$ [6]. The budget constraint for household i amounts to:

$$\left(\widehat{c}(\mu) + t\right) x_i = l_i + T/N + s\Pi^{j}\left(\mu\right). \tag{2}$$

Households are rational and maximize the expected value of utility. When

[5]This can be achieved by assuming that several firms compete *à la* Bertrand by simultaneously setting prices for the polluting good, under the assumption that all firms hold the same beliefs about the environmental damage regardless of whether the observed choice of tax is on or off the equilibrium path. Another way to build this assumption into the model would be to replace the competing firms with a fictitious player called the market.

[6]When s is set strictly lower than $1/N$, it indicates that whereas all the households reside in the policymaker's juridiction, some of the owners of the firms are likely to reside outside her juridiction.

they decide on the consumption of polluting good, they are assumed to take into account the adverse effect of their personal polluting consumption on environmental quality. Denoting $x_{-i} = \Sigma_{j \neq i} x_j$, household i chooses x_i to maximize

$$u(x_i) - \widehat{\varepsilon}(\mu)(\frac{x_i + x_{-i}}{N}) - (\widehat{c}(\mu) + t)\, x_i + T/N + s\Pi^j\,(\mu)\,. \tag{3}$$

The first-order condition for utility maximization is

$$u'(x_i) - \frac{\widehat{\varepsilon}(\mu)}{N} = \widehat{c}(\mu) + t. \tag{4}$$

The right-hand side of (4) gives the marginal rate of substitution between leisure and the polluting good. It must be equal to the relevant marginal rate of transformation, that is, the expected marginal cost of producing the polluting good plus the environmental tax. By solving equation (4), we can write the individual demand for the polluting good as a function of the price to household i and his belief that the policymaker is brown, that is, $X_i(t, \mu)$. Assume that there exists a maximum polluting tax $\bar{t}_\mu$ inside $\mathcal{T}$ such that $u'(0) = \widehat{c}(\mu) + \frac{\widehat{\varepsilon}(\mu)}{N} + \bar{t}_\mu$.

Differentiating (4) yields the following partial derivatives for all $t \in \mathcal{T}$

and $\mu \in [0,1]$:

$$X_t(t,\mu) = \frac{1}{u''(x_i)},$$

$$X_\mu(t,\mu) = \frac{c(\varepsilon^b) - c(\varepsilon^g) + \left(\varepsilon^b - \varepsilon^g\right)/N}{u''(x_i)}, \tag{5}$$

where subscripts denote partial derivatives.

Hence, each demand function is non increasing with tax and, for N sufficiently large, increasing with the probability that the policymaker is brown. Households consume more when they believe the policymaker to be brown because they perceive marginal cost of producing the polluting good to be low. Consequently, when the true type of policymaker is ε^g and households believe her type to be ε^b with probability $\mu > 0$, there is a "price illusion" in the sense that households think the polluting good to be less costly to produce than what it truly is. To ensure that $X_\mu(t,\mu) > 0$, we shall assume henceforth that

$$\forall \varepsilon \in \left[\varepsilon^g, \varepsilon^b\right], c'(\varepsilon) + 1/N < 0. \tag{6}$$

The meaning of inequality (6) is that, in our economy, the individual marginal willingness to pay for pollution reduction $1/N$ is lower than the marginal cleanup cost $-c'(\varepsilon)$, whatever the households' perceptions of the policymaker's concern for the environment.

Let $v_i(t,\mu)$ denote household i's indirect utility function gross of the pollution damage caused by the others' consumption, given subjective beliefs μ:

$$v_i(t,\mu) \equiv u(X_i(t,\mu)) - \widehat{\varepsilon}\left(\mu\right) X_i(t,\mu)/N - \left(\widehat{c}(\mu) + t\right) X_i(t,\mu) + T/N + s\Pi^j\left(\mu\right). \tag{7}$$

Market equilibrium for the polluting good is such that total output must equal the sum of individual consumptions:

$$\Sigma_{i=1}^N l_i = \widehat{c}(\mu)\Sigma_{i=1}^N X_i(t,\mu). \tag{8}$$

Let us consider the first best outcome when households buy a positive quantity of the dirty good. A policymaker of type j aims to maximize welfare measured by $\Sigma_{i=1}^N U(x_i, l_i, E(\overline{x}, \varepsilon^j))$ subject to the decentralized optimizing behavior of households. Hence, the policymaker's objectives reflect the households' preferences and beliefs. Treating all individuals identically, we can omit subscript i and rewrite market equilibrium from (8) as:

$$N\widehat{c}(\mu)X(t,\mu) = Nl. \tag{9}$$

Following Atkinson and Stiglitz (1980, chap 16, p. 493), the policymaker budget constraint can be obtained by summing the individual budget constraints (2) and subtracting the market clearing condition (9):

$$NtX(t,\mu) = T + Ns\Pi^{j}(\mu), \tag{10}$$

where $\Pi^{j}(\mu) = [\widehat{c}(\mu) - c(\varepsilon^{j})]\, X(t,\mu)$. This determines the amount of lump-sum transfers $T^{j}(\mu) \equiv NtX(t,\mu) - Ns\Pi^{j}(\mu)$ which will be handed back to households by the policymaker of type ε^{j} perceived to be ε^{b} with probability μ. Hence, when $\Pi^{j}(\mu) < 0$, revenues from the environmental tax are used to finance losses in profit due to households' misperceptions.

Using (9), the social welfare when the policymaker's type is ε^{j} and is perceived by households to be ε^{b} with probability μ upon seeing the tax t, can be written in the following reduced form function $W(t,\varepsilon^{j},\mu) : \mathcal{T} \times \{\varepsilon^{g},\varepsilon^{b}\} \times [0,1] \rightarrow [0,+\infty)$:

$$\begin{aligned} W(t,\varepsilon^{j},\mu) &\equiv NU(X(t,\mu), \widehat{c}(\mu)X(t,\mu), E\left(X(t,\mu),\varepsilon^{j}\right)) && (11) \\ &= N\left[u\left(X(t,\mu)\right) - \widehat{c}(\mu)X(t,\mu) - \varepsilon^{j}X(t,\mu)\right] && (12) \end{aligned}$$

The expression given in (12) shows that social welfare has two components: first, private welfare $u\left(X(t,\mu)\right) - \widehat{c}(\mu)X(t,\mu)$ which depends on households' perception of the policymaker's concern for the environment, and second, environmental welfare $-\varepsilon^{j}X(t,\mu)$ which directly depends on the true policymaker's concern for the environment.

Welfare functions $W(t, \varepsilon^j, \mu)$ can be shown to satisfy the following single-crossing property which will prove crucial in much of the subsequent analysis:

$$W_{t\varepsilon^j}(t, \varepsilon^j, \mu) > 0. \tag{13}$$

Proof :

Using the expression of indirect utility given in (7), social welfare can be written as

$$W(t, \varepsilon^j, \mu) \quad = \quad N \left[v(t, \mu) + \frac{\widehat{\varepsilon}(\mu)}{N} X(t, \mu) - \varepsilon^j X(t, \mu) + tX(t, \mu) - T/N - s\Pi^j(\mu) . \right] \tag{14}$$

Differentiating $W(t, \varepsilon^j, \mu)$ with respect to t yields

$$W_t(t, \varepsilon^j, \mu) = N \left[\left(t + \frac{\widehat{\varepsilon}(\mu)}{N} - \varepsilon^j \right) X_t(t, \mu) + X(t, \mu) + v_t(t, \mu) \right]. \tag{15}$$

From (7) and the envelope theorem, $v_t(t, \mu) = -X(t, \mu)$ and so

$$W_t(t, \varepsilon^j, \mu) = N \left(t + \frac{\widehat{\varepsilon}(\mu)}{N} - \varepsilon^j \right) X_t(t, \mu). \tag{16}$$

As $X_t(t, \mu) < 0$ (see (5)), $W_{t\varepsilon^j}(t, \varepsilon^j, \mu) = -NX_t(t, \mu) > 0$. **Q.E.D.**

In the present context, the single-crossing property (13) guarantees that the brown policymaker is more inclined than her green counterpart to increase tax because the resulting decrease in consumption reduces more polluting wastes. A straightforward consequence is that the brown policymaker has a greater incentive for using environmental tax as a signaling device that informs households about her true concern for the environment.

3 The discretionary tax policy

To see the effect of households' misperception of the policymaker's concern for the environment, consider the situation in which households make their consumption decision without knowledge of the policymaker's choice of tax, say because the policymaker cannot make binding commitments on the level of the environmental tax. When environmental tax has no commitment value, the policymaker's behavior provides no information on her environmental concern and households' inferences are consequenceless. This makes relevant to use the standard concept of a Bayesian equilibrium. Under a discretionary regime of environmental tax, a (pure-strategy) Bayesian equilibrium is a set of strategies $\{(t^j(\mu_0))_{j=b,g}, (X_i(\mu_0))_{i=1,2...,N}\}$ such that :

$$\text{For } j = b, g, t^j(\mu_0) \in \arg\max_{t_j} W(t_j, \varepsilon^j, \mu_0), \tag{17}$$

$$\begin{aligned} \text{for } i &= 1, ..., N, \\ X_i(\mu_0) &\in \arg\max_{x_i} \left[\begin{array}{c} u(x_i) - \widehat{\varepsilon}(\mu_0)\left(\frac{x_i + x_{-i}}{N}\right) \\ -\left(\mu_0(c(\varepsilon^b) + t^b(\mu_0)) + (1-\mu_0)(c(\varepsilon^g) + t^g(\mu_0))\right) x_i + T/N + s\Pi. \end{array} \right] \end{aligned} \tag{18}$$

Lemma 1 states the optimal values of the environmental tax under a discretionary regime, to the extent that problem (17) admits an interior solution. It turns out that $\varepsilon^g - \frac{\varepsilon^b}{N} > 0$ is a sufficient condition for $t^j(\mu_0)$ to be an interior solution whatever $\mu_0 \in [0,1]$ and $j = b, g$.

Lemma 1: *For $j = b, g$, the tax $t^j(\mu_0) \in \left[0, \bar{t}_{\mu_0}\right]$ that implements the socially optimal outcome without commitment is given by:*

$$t^j(\mu_0) = \max\left\{0, \varepsilon^j - \frac{\widehat{\varepsilon}(\mu_0)}{N}\right\}.$$

If $\varepsilon^g - \frac{\varepsilon^b}{N} > 0$, then for any $\mu_0 \in [0,1]$ and $j = b, g$, $t^j(\mu_0) > 0$.

Proof :

From (16), the necessary condition for optimality is $t + \frac{\widehat{\varepsilon}(\mu_0)}{N} - \varepsilon^j = 0$ for a solution inside $\left[0, \bar{t}_{\mu_0}\right]$. Solving this condition for t yields the discretionary tax $t^j(\mu_0)$. The second-order condition for welfare maximization when evaluated at the optimum yields $W_{tt}(t, \varepsilon^j, \mu_0) = X_t(t, \mu_0)$

which is negative (see (5)), due to both the assumptions of quasi-linear preferences and the concavity of $u(x)$. **Q.E.D**.

When the pollution tax has no commitment value, the optimal discretionary tax is equal to the marginal environmental damage ε^j less the part of pollution $\frac{\widehat{\varepsilon}(\mu_0)}{N}$ that is internalized by each household given his prior probability that the policymaker is brown. This is reminiscent of Diamond's (1973) result that the optimal tax is a weighted average of the externalities, the weight of each household $\frac{\varepsilon^j}{N}$ being corrected for the household's misperception $\widehat{\varepsilon}(\mu_0)$. If households' perceptions were accurate, then $\widehat{\varepsilon}(\mu_0) = \varepsilon^j$ and the optimal discretionary tax would be $\frac{N-1}{N}\varepsilon^j$, as in Diamond (1973). A common assumption in the public economics literature is that households do not take into account the adverse effect of their dirty consumption on the environment (see, for instance, Bovenberg and van der Ploeg (1994)). Such an assumption would impose here that N approaches infinity. Suppose that households ignore environmental externalities and perfectly know the policymaker's concern for the environment, then the optimal discretionary tax is simply the Pigovian level ε^j that fully internalizes the marginal environmental damage (see, for comparison, equation (6) in Bovenberg and de Mooij (1994) to the extent that the marginal utility of income in the present context is given by $U_l(x, l, E) = -1$). In some sense, the expression of $t^j(\mu_0)$ in Lemma 1 involves a “fiscal illusion” in the spirit of Filimon et alii (1982), where this notion is restricted here to some difference in beliefs between households and bureaucrats about the environmental damage.

Interestingly enough, the present model satisfies a property of stochastic dominance, at least for values of the tax higher than the optimal discretionary tax, which is known to be essential for whom is familiar with standard signaling games.

Property 1: For all $\varepsilon^j \in \{\varepsilon^g, \varepsilon^b\}$, $\mu \in [0,1]$ and $t \in \left[t^j(\mu), \bar{t}_\mu\right]$, if (6) holds, then $W_\mu(t, \varepsilon^j, \mu) > 0$.

Proof :

From the expression given in (14), differentiating $W(t, \varepsilon^j, \mu)$ with respect to μ yields:

$$W_\mu(t, \varepsilon^j, \mu) = N\left[\left(t + \frac{\widehat{\varepsilon}\,(\mu)}{N} - \varepsilon^j\right) X_\mu(t, \widehat{\varepsilon}) + \frac{X(t,\mu)}{N}\left(\varepsilon^b - \varepsilon^g\right) + v_\mu(t,\mu)\right]. \tag{19}$$

From (7) and the envelope theorem, $v_\mu(t,\mu) = -\left(\left(\varepsilon^b - \varepsilon^g\right)/N + c(\varepsilon^b) - c(\varepsilon^g)\right) X(t,\mu)$. Recall from (5) that

$X_\mu(t,\mu) = \frac{c(\varepsilon^b) - c(\varepsilon^g) + \left(\varepsilon^b - \varepsilon^g\right)/N}{u''(x_i)}$ and so (19) can be rewritten

$$W_\mu(t, \varepsilon^j, \mu) = N\left[\left(t + \frac{\widehat{\varepsilon}\,(\mu)}{N} - \varepsilon^j\right)\frac{c(\varepsilon^b) - c(\varepsilon^g) + \left(\varepsilon^b - \varepsilon^g\right)/N}{u''(x_i)} - (c(\varepsilon^b) - c(\varepsilon^g))X(t,\mu)\right]. \tag{20}$$

As $t^j(\mu) = \varepsilon^j - \frac{\widehat{\varepsilon}(\mu)}{N}$, it follows from (20) that, for all $t \in \left[t^j(\mu), \bar{t}_{\mu_0}\right]$, $W_\mu(t, \varepsilon^j, \mu) > 0$, provided that condition (6)

holds. **Q.E.D**.

Property 1 states that, given any tax higher than the optimal discretionary tax, social welfare is higher when households believe the policymaker more likely to be brown. An increase in μ has two effects. First, under Assumption (6), it raises the demand for the polluting good. In equilibrium, taxes are such that this effect on welfare is equal to zero. Second, by changing μ, the policymaker may influence households' perception of the polluting good price. An increase in μ makes the polluting good more attractive by modifying households' expectations on $\widehat{c}(\mu)$, thereby raising the private component of social welfare (see (12)). Consequently, if households infer from any deviation of the pollution tax above the optimal discretionary level $t^j(\mu)$ that the policymaker is brown with probability μ, then the green policymaker may have an incentive to withhold the truth from households. Consider in particular the case in which households ignore environmental externalities, that is, $1/N$ is null. Suppose further that information is perfect and the policymaker is green, then it is optimal for the latter to set the Pigovian tax ε^g which fully internalizes the marginal environmental damage. Given this tax however, the true information, i. e. $\mu = 0$, is the worst belief, as shown by $W_\mu(\varepsilon^g, \varepsilon^g, \mu) = -N(c(\varepsilon^b) - c(\varepsilon^g))X(t = \varepsilon^g, \mu) > 0$. Indeed, social welfare would be higher if environmental damage were to be overestimated by households, that is, for any μ inside $(0,1]$. The illusion that the price of the polluting good $\widehat{c}(\mu)$ is lower than $c(\varepsilon^g)$ boosts the private component of social welfare.

Stochastic dominance is an essential structural property of standard signaling games[7]. Therefore, we shall extend Property 1 to the whole set of taxes:

$$\text{for all } \varepsilon^j \in \{\varepsilon^g, \varepsilon^b\}, \mu \in [0,1] \text{ and } t \in \left[0, \bar{t}_1\right], W_\mu(t, \varepsilon^j, \mu) > 0. \tag{21}$$

Hence, for any t, $\mu = 0$ is the least favorable belief for the policymaker, whatever her environmental concern. Figure 1 depicts possible shapes for the social welfare functions $W(t, \varepsilon^j, \mu)$, for $j = b, g$ and $\mu = 0, 1$.

4 The enforceable tax policy

Let us now consider that commitments are feasible through legal arrangements or other procedures, so that environmental tax is enforceable. Then, the policymaker's choice of environmental tax can now be observed by households before they make their consumption decision. This gives the model a structure of signaling game, for which strategies must form a perfect Bayesian

[7]This property is implied by assumptions A1', A2 and A3 taken together page 392 in Cho and Sobel (1990) or directly assumed page 255 in Mailath, Okuno-Fujiwara and Postlewaite (1993.)

equilibrium. As suggested by Harsanyi (1967-8), such a game of incomplete information can be replaced by a game of complete but imperfect information which unfolds in three stages. First, "Nature" draws a type ε^j of policymaker from the set $\{\varepsilon^g, \varepsilon^b\}$ according to the probability distribution μ_0. Second, the policymaker learns her environmental preferences and imposes the tax on the polluting good. After observing this tax, households in the third stage rely on their inferences upon the true value of ε^j to make their consumption decision. Let $\mu(t) : (0,1) \times \mathcal{T} \to [0,1]$ denote the households' posterior belief that the policymaker is of type ε^b which updates the prior μ_0 when the tax is t. We assume that, after observing tax choice t, households and firms hold the same belief about the policymaker's concern for the environment and define $\widehat{c}(\mu(t)) \equiv \mu(t)c(\varepsilon^b) + (1-\mu(t))c(\varepsilon^g)$ as the expected cost of producing the polluting good after observing t. The policymaker, in turn, must take into account how her choice of tax influences households' inferences. Restricting attention to pure strategies, a perfect Bayesian equilibrium of this game is a set of strategies $\left\{(t_j^*)_{j=b,g}, (X_i^*(t, \widehat{\varepsilon}(\mu^*(t))))_{i=1,2\ldots,N}\right\}$ and a probability distribution $\mu^*(t)$ such that, at any stage of the game, strategies must be optimal given beliefs:

Condition 1: optimality condition for the policymaker.

$$\text{For } j = b, g, t_j^* \in \arg\max_{t_j} W(t_j, \varepsilon^j, \widehat{\varepsilon}(\mu^*(t_j))).$$

Condition 2: perfection condition for the households.

For all t_j and $i = 1, ..., N$,

$$X_i^*(t_j, \widehat{\varepsilon}(\mu^*(t_j))) \in \arg\max_{x_i} \left[\begin{array}{c} u(x_i) - \widehat{\varepsilon}(\mu^*(t_j))\,(\frac{x_i + x_{-i}}{N}) \\ -(\widehat{c}(\mu^*(t_j)) + t_j)\,x_i + T/N + s\Pi \end{array} \right].$$

Condition 3: Bayes' consistency of beliefs on the equilibrium path.

$$\text{If } t_g^* \neq t_b^*, \text{ then } \mu^*(t_g^*) = 0 \text{ and } \mu^*(t_b^*) = 1;$$
$$\text{If } t_g^* = t_b^*, \text{ then } \mu^*(t_g^*) = \mu^*(t_b^*) = \mu_0.$$

Condition 1 demands that the policymaker's choice of tax maximizes social welfare given that households respond optimally. Condition 2 states that households' consumption of the polluting good should maximize their utility given their beliefs induced by the policymaker's behavior. Finally, condition 3 requires the households' posterior beliefs about ε^j to be formed from their prior beliefs by using Bayes' rule for the policymakers' equilibrium strategies. As usual, households are assumed to revise their beliefs in an arbitrary way off the equilibrium path.

4.1 Separating and pooling equilibria

Our interest now is not really a characterization of all perfect Bayesian equilibria in the model but rather a characterization of the set of perfect Bayesian equilibrium taxes. From assumption (21), we can without loss of generality let the households' beliefs be $\mu(t) = 0$ for all $t \notin \{t_g^*, t_b^*\}$; thus the households'out-of-equilibrium beliefs are always the least favorable for the policymaker. Such beliefs are the strongest too in that, if a policymaker of any type does not have an incentive to set t when $\mu(t) > 0$, then she will not have an incentive when $\mu(t) = 0$, since social welfare is lower. Therefore setting $\mu(t) = 0$ will generate all of the possible perfect Bayesian equilibrium paths.

Let t_g^* and t_b^* denote the separating equilibrium taxes when the policymaker is green and brown, respectively. The corresponding equilibrium welfares are W_g^* and W_b^*. The best choice for the green policymaker is to set t_g^* equal to the symmetric information Pigovian level that fully internalizes the environmental damage when households' perceptions of this damage are accurate, i. e., $t^g(0) = \varepsilon^g - \frac{\varepsilon^g}{N}$.

Lemma 2: *If* (21) *holds and* $\mu(t) = 0$ *for all* $t \notin \{t_g^*, t_b^*\}$, *then in any separating equilibrium* $t_g^* = t^g(0)$.

Proof :

Suppose for a contradiction that there exists a separating equilibrium in which $t_g^* \neq t^g(0)$. As households'expectations are correct at equilibrium,

the resulting social welfare is $W(t, \varepsilon^g, 0)$ which is strictly lower than $W(t^g(0), \varepsilon^g, 0)$. Then, the policymaker would have an incentive to deviate to

$t^g(0)$ whatever the households' inference μ from observing $t^g(0)$. Indeed, for any $\mu \in (0, 1]$, we have

$$W(t^g(0), \varepsilon^g, 0) = W(t^g(0), \varepsilon^g, \mu) + \int_\mu^0 W_\mu(t^g(0), \varepsilon^g, \rho) d\rho.$$

From Property 1, $W_\mu(t^g(0), \varepsilon^g, \mu) > 0$, thus, $\int_\mu^0 W_\mu(t^g(0), \varepsilon^g, \rho) d\rho < 0$ and $W(t^g(0), \varepsilon^g, 0) < W(t^g(0), \varepsilon^g, \mu)$. If $t_g^* \neq t^g(0)$, then

$W(t_g^*, \varepsilon^g, 0) < W(t^g(0), \varepsilon^g, 0)$ and so $W(t_g^*, \varepsilon^g, 0) < W(t^g(0), \varepsilon^g, \mu)$. It turns out that $t_g^* = t^g(0)$ can be supported as the only separating

equilibrium tax for the green policymaker, given the assumption $\mu(t) = 0$ for all $t \neq t_g^*$ since $W(t, \varepsilon^g, 0) < W(t^g(0), \varepsilon^g, 0)$.**Q.E.D**.

Consider now situations where households believe they know the true environmental concern of the policymaker. If households assign probability 1 to the brown policymaker while the policymaker is in fact green, demand for the polluting good after observing some tax t is $X(t, 1)$ and social welfare is given by $W(t, \varepsilon^g, 1)$. On the other hand, if households are wrongly convinced that the policymaker is green, demand and social welfare are respectively $X(t, 0)$ and $W(t, \varepsilon^b, 0)$. Setting the optimal discretionary tax $t^b(0)$ would yield a welfare of $W(t^b(0), \varepsilon^b, 0)$, which can be interpreted as the opportunity cost for the brown policymaker to fully transmit information on her environmental concern.

To achieve separation, the brown policymaker must choose an environmental tax t_b^* that satisfies the two following conditions:

$$W(t_b^*, \varepsilon^g, 1) \;\; \leq \;\; W(t^g(0), \varepsilon^g, 0) \tag{22}$$

$$W(t_b^*, \varepsilon^b, 1) \;\; > \;\; W(t^b(0), \varepsilon^b, 0). \tag{23}$$

Condition (22) is an incentive compatibility (IC) constraint ensuring that the green policymaker has no incentive to lie about her true environmental concern. She must choose the symmetric information Pigovian tax and fully reveal information on her concern for the environment rather than trick households into believing that she is brown by setting t_b^*. As can be seen in Figure 1, the IC constraint defines a set $\mathcal{T}_g$ of possible taxes t_b^* that satisfy (22). Note that assumption (6) implies that $\bar{t}_0 \equiv u'(0) - c(\varepsilon^g) - \frac{\widehat{\varepsilon}(0)}{N} < \bar{t}_1 \equiv u'(0) - c(\varepsilon^b) - \frac{\widehat{\varepsilon}(1)}{N}$. The equality version of (22) admits an upper root which will be denoted by $\bar{t}_g \equiv \max\left\{t \in \left[0, \bar{t}_1\right] / W(t, \varepsilon^g, 1) = W(t^g(0), \varepsilon^g, 0)\right\}$. It will be shown that the tax $\bar{t}_c$ set by the brown policymaker corresponds in some circumstances to the so-called "least-cost separating equilibrium" which has received much emphasis in the work of Spence (1974), Riley (1979) and Cho-Kreps (1987), among others.

The second condition (23) is an individual rationality (IR) constraint guaranteeing that the brown policymaker would rather charge t_b^* and transmit all information than let households wrongly perceive her as green and optimize accordingly. Let $\mathcal{T}_b$ denote the interval of taxes for which condition (23) is met (see Figure 1) and define $\bar{t}_b \equiv \max\left\{t \in \left[0, \bar{t}_1\right] / W(t, \varepsilon^b, 1) = W(t^b(0), \varepsilon^b, 0)\right\}$.

In order to fully reveal that environment is dirty, the policymaker must choose t_b^* in the interval $\mathcal{T}_g \cap \mathcal{T}_b$ provided that the latter is non empty. This interval is depicted in Figure 1. Proposition 1 demonstrates the existence of a class of separating equilibria.

Proposition 1: *If* (21) *and* (13) *hold, and* $\mu(t) = 0$ *for all* $t \notin \left\{t_g^*, t_b^*\right\}$, *then there exists a continuum of separating equilibria in which the green policymaker sets* $t_g^* = t^g(0)$ *and the brown policymaker sets* $t_b^* \in [\bar{t}_c, \bar{t}_d)$.

Proof :

Using the definitions of $\bar{t}_g$ and $\bar{t}_b$, we have

$$W(\bar{t}_g, \varepsilon^g, 1) = W(t^g(0), \varepsilon^g, 0) \tag{24}$$

$$W(\bar{t}_b, \varepsilon^b, 1) = W(t^b(0), \varepsilon^b, 0). \tag{25}$$

Subtracting (24) from (25) yields

$$\int_{\bar{t}_g}^{\bar{t}_b}\int_{\varepsilon^g}^{\varepsilon^b} W_{t\varepsilon}(t,\varepsilon,1)d\varepsilon dt = \int_{t^g(0)}^{t^b(0)}\int_{\varepsilon^g}^{\varepsilon^b} W_{t\varepsilon}(t,\varepsilon,0)d\varepsilon dt. \tag{26}$$

From Lemma 1, $t^b(0) = \varepsilon^b - \frac{\varepsilon^g}{N} > t^g(0) = \varepsilon^g - \frac{\varepsilon^g}{N}$. Moreover, the single-crossing property (13) guarantees that, for all $\mu \in [0,1]$,

$W_{t\varepsilon}(t,\varepsilon,\mu) > 0$. Hence, equality (26) implies that

$$\int_{\bar{t}_g}^{\bar{t}_b}\int_{\varepsilon^g}^{\varepsilon^b} W_{t\varepsilon}(t,\varepsilon,1)d\varepsilon dt > 0, \tag{27}$$

and so

$$\bar{t}_g < \bar{t}_b. \tag{28}$$

An argument similar to that above can show that

$\min\left\{t \in \left[0,\bar{t}_1\right]/W(t,\varepsilon^g,1) \leq W(t^g(0),\varepsilon^g,0)\right\} < \min\left\{t \in \left[0,\bar{t}_1\right]/W(t,\varepsilon^b,1) > W(t^b(0),\varepsilon^b,0)\right\}$

to the extent that at least one of these values is not zero. Thus, $\mathcal{T}_g \cap \mathcal{T}_b = [\bar{t}_g, \bar{t}_b) \neq 0$.

Clearly, any tax $t_b^* \in [\bar{t}_g, \bar{t}_b)$ can be supported as a separating equilibrium for the brown policymaker by out-of-equilibrium beliefs such that

$\mu^*(t) = 0$ for all $t \neq t_b^*$ since $W(t,\varepsilon^b,0) \leq W(\bar{t}_d,\varepsilon^b,1) < W(t_b^*,\varepsilon^b,1)$. **Q.E.D**.

As a result, there always exists an environmental tax that fully informs households about the policymaker's concern for the environment, regardless of whether she is green or brown. The separating equilibrium tax chosen by the green policymaker is uniquely set equal to the symmetric information Pigovian level $t^g(0) = \varepsilon^g - \frac{\varepsilon^g}{N}$. By contrast, there is a whole range of separating equilibrium taxes for the brown policymaker, namely $[\bar{t}_g, \bar{t}_b)$.

Note that if we were to consider the case in which the policymaker's private valuation $\widehat{\varepsilon}$ is a random variable described by a cumulative distribution function and a density with continuum support $\left[\varepsilon^g, \varepsilon^b\right]$, the single-crossing property (13) and the property of stochastic dominance (21) taken together would be sufficient to guarantee the existence of separating equilibria (see Mailath (1987) for a detailed proof).

The next corollary gives sufficient conditions for the taxes signaling the brown policymaker to deviate from the symmetric information Pigovian level. This is precisely the situation for which Figure 1 is drawn. It arises when the parameter values of the model are such that the green policymaker would "envy" the welfare attained by the brown policymaker with her symmetric information Pigovian level, that is, $W(t^b(1), \varepsilon^g, 1) > W(t^g(0), \varepsilon^g, 0)$.

Corollary 1: *Assume* (21) *and* (13) *hold. If* $u'(x) > \varepsilon^g + c(\varepsilon^g)$ *and* $X(t^b(1), 1) > X(t^g(0), 0)$, *then* $t_b^* > t^b(1)$.

Proof :

If $u'(x) > \varepsilon^g + c(\varepsilon^g)$ and $X(t^b(1), 1) > X(t^g(0), 0)$, then

$\int_{X(t^g(0),0)}^{X(t^b(1),1)} [u'(x) - (\varepsilon^g + c(\varepsilon^g))]\,dx > 0$. Thus, $u\left(X(t^b(1),1)\right) - u\left(X(t^g(0),0)\right) > (\varepsilon^g + c(\varepsilon^g))\left(X(t^b(1),1) - X(t^g(0),0)\right)$.

Moreover, on one hand we have

$W(t^b(1),\varepsilon^g,1) = u\left(X(t^b(1),1)\right) - \varepsilon^g X(t^b(1),1) - c\left(\varepsilon^b\right) X(t^b(1),1) > u\left(X(t^b(1),1)\right) - (\varepsilon^g + c(\varepsilon^g)) X(t^b(1),1)$; on the other hand, we have

$W(t^g(0),\varepsilon^g,0) = u\left(X(t^g(0),0)\right) - (\varepsilon^g + c(\varepsilon^g)) X(t^g(0),0)$.

As a result, $W(t^b(1),\varepsilon^g,1) > W(t^g(0),\varepsilon^g,0)$. Thus, $t^b(1)$ violates the IC condition for separation given by (24), hence $t^b(1) < \bar{t}_g$.

Q.E.D.

The condition that $u'(x) > \varepsilon^g + c(\varepsilon^g)$ means that the marginal utility from consuming the polluting good is higher than the marginal social cost of producing this good when the policymaker is green. If moreover the socially optimal demand is higher with the brown policymaker than with the green one, that is, $X(t^b(1),1) > X(t^g(0),0)$, then the green policymaker has all reasons to envy her brown counterpart in the sense that $W(t^b(1),\varepsilon^g,1) > W(t^g(0),\varepsilon^g,0)$. In such circumstances, the green policymaker might be tempted to set the high tax $t^b(1)$ if doing so could trick households into believing she is brown. As previously seen, this would boost the private component of social welfare. To prevent such a masquerade, the brown policymaker must suffer a welfare loss too high to be judged worthwhile by the green policymaker. As a result, the taxes signaling the brown policymaker are set above the symmetric information Pigovian level, i. e.,

$t^b(1) = \varepsilon^b - \frac{\varepsilon^b}{N}$. In such a case, the policymaker reveals she is brown by overtaxing relative to the level that would fully internalize the environmental damage were households' perceptions of this damage the same as the policymaker's perception. Hence, signaling a high environmental damage may entail a welfare sacrifice to tell households the truth about the actual policymaker's concern for the environment: the policymaker must reduce consumption even more than what would prevail if households and the policymaker were to have the same information about environmental damage. A straightforward consequence is that the amount of pollution associated with t_b^* when $W(t^b(1), \varepsilon^g, 1) > W(t^g(0), \varepsilon^g, 0)$, i. e., $\varepsilon^b X(t_b^*, 1)$, must be lower than the amount of pollution obtained with the symmetric information Pigovian tax, i. e., $\varepsilon^b X(t^b(1), 1)$.

The tax can transmit all information on the policymaker's concern for the environment because, from the single-crossing property stated in (13), increasing tax is more damaging to the green policymaker than it is to the brown policymaker. This implies that the brown policymaker may reduce consumption and curb pollution more than what would be implied by the Pigovian principle under symmetric information.

The existence of separating equilibria does not dismiss pooling equilibria. Let t^* denote the uninformative tax that gives a pooling equilibrium. Since it is the same tax levied by the policymaker whatever her environmental concern, the households' posterior beliefs after observing the tax t^* are the same as their prior beliefs. To be part of a pooling equilibrium, t_g^* must satisfy the two following conditions:

$$W(t_g^*, \varepsilon^j, \mu_0) \geq W(t^j(0), \varepsilon^j, 0), j = b, g. \tag{29}$$

What (29) says is that the policymaker would rather impose t_g^* and conceal information on her true environmental concern than make households be sure that she is green and optimize accordingly.

Define $\bar{t}_g(\mu_0) \equiv \max\left\{t \in \left[0, \bar{t}_1\right] / W(t, \varepsilon^g, \mu_0) = W(t^g(0), \varepsilon^g, 0)\right\}$ and $\underline{t}_b(\mu_0) \equiv \min\left\{t \in \left[0, \bar{t}_1\right] / W(t, \varepsilon^b, \mu_0) \geq W(t^b(0), \varepsilon^b, 0)\right\}$ (see Figure 2). Integrating and using the single-crossing property along the same line as for the proof of Proposition 1, it can easily be shown that $\underline{t}_b(\mu_0) < \bar{t}_g(\mu_0)$. The next proposition characterizes the set of pooling equilibrium taxes, which is depicted in Figure 2.

Proposition 2: *If* (21) *and* (13) *hold, then any tax* $t^* \in \left[\underline{t}_b(\mu_0), \bar{t}_g(\mu_0)\right]$ *can be supported as a pooling equilibrium by beliefs* $\mu^*(t) = 0$ *for all* $t \neq t^*$.

However, many pooling equilibria can be argued to involve unreasonable beliefs to the extent that they are not robust to standard refinement criterions. Many separating equilibria can be discarded in the same way, as will be shown in the next section.

4.2 Equilibrium selection

The logic of *Undefeated Equilibrium* (UE) proposed by Mailath, Okuno-Fujiwara and Postlewaite (1993) will be applied here to eliminate all the equilibria, separating and pooling, which are Pareto dominated by other equilibria. The main reason for emphasizing undefeated equilibria here is to formalize the idea that the policymaker ought to be able to influence households' beliefs because she is in position of relative power. Before learning the actual damage of the environment, the policymaker could commit to restrict herself to the set of UE taxes when levying environmental taxes, and so influence households' expectations.

Define W_j^* and $W_j^{\#}$ as the equilibrium social welfares when the policymaker of type ε^j imposes taxes t_j^* and $t_j^{\#}$ respectively. Furthermore, let $\mu^*(t)$ be the posterior belief held by consumers after observing t, that sustains t_j^* as an equilibrium. Following Mailath, Okuno-Fujiwara and Postlewaite (1993), the equilibrium involving the tax t_j^* survives the UE criterion, provided that there does not exist another equilibrium with $t_j^{\#}$ "defeating" the equilibrium with t_j^*, i. e.:

$$\begin{aligned}
\text{If } t_g^{\#} &= t_b^{\#}, && (30)\\
\text{then } W_j^{\#} &\geq W_j^* \text{ for } j = b, g \text{ with one inequality strict and } \mu^*\left(t_b^{\#}\right) \neq \mu_0. \\
\text{If } t_g^{\#} &\neq t_b^{\#}, && (31)\\
\text{then } W_b^{\#} &> W_b^*, W_g^{\#} \leq W_g^* \text{ and } \mu^*\left(t_b^{\#}\right) \neq 1.
\end{aligned}$$

The underlying idea is that the existence of an equilibrium with $t_j^{\#}$ would induce the policymaker j to deviate from t_j^* since it would yield at least as much welfare. Requirement (31) is similar to that imposed by Cho and Kreps (1987) to meet the "intuitive criterion". Some persuasive arguments in favor of the UE criterion can be found in Mailath, Okuno-Fujiwara and Postlewaite (1993). Note that using this criterion for signaling games is in the same spirit as requiring an allocation to be renegociation-proof in an informed principal's model with common values (see the concept of *Weakly Interim Efficiency* in Maskin and Tirole (1992)).

To refine the set of equilibrium taxes, we shall focus on the most interesting case where $W(t^b(1), \varepsilon^g, 1) > W(t^g(0), \varepsilon^g, 0)$, i. e., the only way for a policymaker to reveal her environmental concern is to distort the environmental tax above the symmetric information Pigovian level $t^b(1)$. (See Corollary 1 for the conditions under which the previous inequality is satisfied)

Proposition 3: *If* $u'(x) > \varepsilon^g + c(\varepsilon^g)$ *and* $X(t^b(1), 1) > X(t^g(0), 0)$, *then all the separating equilibrium taxes are defeated by the following pair of least-costly separating equilibrium taxes:*

- $t_g^* = t^g(0)$ *for the green policymaker,*
- *and* $t_b^* = \bar{t}_g$ *for the brown policymaker.*

Associated with these taxes, the damages from pollution are respectively:

- $\varepsilon^g X(t^g(0), \varepsilon^g)$ *with the green policymaker, which is the same as that allowed by the symmetric information Pigovian tax;*
- $\varepsilon^b X(\bar{t}_c, \varepsilon^b)$ *with the brown policymaker, which is lower than that allowed by the symmetric information Pigovian tax.*

Proof :

Assume $u'(x) > \varepsilon^g + c(\varepsilon^g)$ and $X(t^b(1), 1) > X(t^g(0), 0)$. Suppose that separation is achieved in equilibrium at $t_b^* > \bar{t}_g$. From Corollary 1, we

have $\bar{t}_g > t^b(1)$, hence $W(t_b^*, \varepsilon^b, 1) < W(\bar{t}_g, \varepsilon^b, 1)$. The out-of-equilibrium beliefs that support t_b^* as a separating equilibrium price must assign

probability $\mu^*\left(\bar{t}_g\right) < 1$ at $\bar{t}_g$ otherwise the brown policymaker would deviate to $\bar{t}_g$. Then, from (31), the separating equilibrium with t_b^* is

defeated by the separating equilibrium with $\bar{t}_g$. **Q.E.D.**

There are infinitely many separating equilibrium taxes at which the brown policymaker can reveal her environmental concern. However, from requirement (31), the separating equilibrium entailing the least-costly signal defeats all the other separating equilibria. As $W(t^b(1), \varepsilon^g, 1) > W(t^g(0), \varepsilon^g, 0)$, the only way for a brown policymaker to fully reveal her concern for the environment at an environmental tax higher than if she were green, is to distort the tax above the symmetric information Pigovian level $t^b(1)$. Otherwise, the policymaker would find it optimal to impose the high tax even if she were green. Thus, the brown policymaker must reduce consumption even more than what would require the Pigovian principle under symmetric information. By setting $\bar{t}_g$, the policymaker minimizes the welfare loss from reducing consumption. This loss is then measured by the difference $W(t^b(1), \varepsilon^b, 1) - W(\bar{t}_g, \varepsilon^b, 1)$.

Let us now return to the analysis of pooling equilibria. Pooling with a tax t^* is potentially costly for the brown policymaker. The welfare differential relative to the socially optimal situation under symmetric information is then given by $W(t^b(1), \varepsilon^b, 1) - W(t^*, \varepsilon^b, \mu_0)$. Requirement (31) imposes the following restriction on t^*:

$$W(\bar{t}_g, \varepsilon^b, 1) \leq W(t^*, \varepsilon^b, \mu_0). \tag{32}$$

If inequality (32) holds, then social welfare achieved by the brown policymaker with the pooling equilibrium price t^* is higher than with the separating equilibrium price $\bar{t}_g$. As the green policymaker will also prefer to duplicate t^* than to reveal information, requirement (32) guarantees that the pooling equilibrium defeats the least-costly separating equilibrium. According to the logic advocated by Mailath, Okuno-Fujiwara and Postlewaite (1993), it is more "plausible" that, in such a case, the policymaker will choose to provide no information in equilibrium. Clearly, condition (32) will hold for sufficiently high values of μ_0. More precisely, let $\overline{\mu}$ define the unique probability μ such that

$$W(\bar{t}_g, \varepsilon^b, 1) = W(t^b(\mu), \varepsilon^b, \mu). \tag{33}$$

The right-hand side of equality (33) is the socially optimal welfare under a discretionary regime when the policymaker is brown and is thought at the same time to be brown with probability μ: this is the maximum that the brown policymaker can obtain by holding back information. Consider Figure 3. The critical value $\overline{\mu}$ is the level of beliefs such that the brown policymaker is indifferent between revealing her concern for the environment at the least cost and concealing information about it. In other words, when households assign probability $\overline{\mu}$ to the brown policymaker, the latter is indifferent between fully revealing information with a tax $\bar{t}_g$ or providing no information with $t^b(\overline{\mu})$. If μ_0 is strictly lower than $\overline{\mu}$, then inequality (32) is violated whatever the uninformative tax t^* considered. Consequently, the policymaker is expected to transmit, in equilibrium, all information on her environmental concern. The result is reversed for values of μ_0 higher than $\overline{\mu}$. Such "pessimistic" beliefs held by households about the environmental concern of the policymaker do not allow for the policymaker's choice of tax to fully reveal information because these beliefs increase the opportunity cost of fully revealing information for the brown policymaker. Instead, the policymaker will choose to conceal information by choosing the same tax in equilibrium whatever her type.

Recall from Lemma 1 that $t^g(\mu_0) = \max\left\{0, \varepsilon^g - \frac{\widehat{\varepsilon}(\mu_0)}{N}\right\} \leq t^b(\mu_0) = \max\left\{0, \varepsilon^b - \frac{\widehat{\varepsilon}(\mu_0)}{N}\right\}$. This implies that, for all $t^* \in [\underline{t}_b(\mu_0), t^g(\mu_0)) \cup (t^b(\mu_0), \overline{t}_g(\mu_0)]$,

$$W(t^*, \varepsilon^j, \mu_0) < W(t^j(\mu_0), \varepsilon^j, \mu_0), j = b, g. \tag{34}$$

Thus, all the pooling equilibria involving a price either strictly lower than $t^g(\mu_0)$ or strictly higher than $t^b(\mu_0)$ and a prior probability $\mu_0 \in (\overline{\mu}, 1)$, are defeated by the pooling equilibrium involving the price $t^j(\mu_0)$ and the prior probability μ_0. Figure 3 illustrates this case. Finally, Proposition 4 presents all the equilibria surviving the refinement criterion proposed by Mailath, Okuno-Fujiwara and Postlewaite (1993).

Proposition 4:

Assume that $u'(x) > \varepsilon^g + c(\varepsilon^g)$ and $X(t^b(1), 1) > X(t^g(0), 0)$.

1. *If $\mu_0 \leq \overline{\mu}$, then separation is achieved with the pair of UE taxes $t_g^* =$*

$t^g(0)$ and $t_b^* = \bar{t}_g$.

2. *If $\mu_0 > \overline{\mu}$, then a continuum of UE exists, which is characterized by pooling at any tax inside $\left[t^g(\mu_0), t^b(\mu_0)\right]$.*

The selection of UE taxes allows to mitigate the current concentration on the least-costly separating equilibrium, which involves the taxes $t_g^* = t^g(0)$ and $t_b^* = \bar{t}_g$ in the present case, and turn attention to pooling equilibrium taxes, which are no less economically intuitive.

On the one hand, when households a priori believe less likely the policymaker to be brown, then the latter must impose a welfare loss to households in order to fully reveal that she is actually brown. In such a case, the environmental tax is distorted above the symmetric information Pigovian level. Households incur a signaling cost to learn that the polluting good is cheaper than they initially thought. Obviously, the pollution level that results from the environmental tax levied by the brown policymaker falls below the symmetric information Pigovian level. Overtaxation and underpollution with the brown policymaker are necessary to remove the incentive for the green policymaker to lie about her environmental concern.

On the other hand, when households a priori believe more likely the policymaker to be brown, the emergence of separating equilibrium taxes is dubious to the extent that telling the truth has become more costly for the brown policymaker than concealing her true environmental concern from households. Then, the policymaker can be expected to provide no information on her true environmental concern and pooling equilibria will prevail. In any such undefeated pooling equilibrium, the brown (resp. green) policymaker would rather distort the environmental tax below (resp. above) the Pigovian tax, which fully internalizes the marginal social damage from pollution under symmetric information. As a result, when the incentive for the green policymaker to lie about her environmental concern fails to be removed in equilibrium, there is more pollution with the brown policymaker and less pollution with the green policymaker relative to what the Pigovian principle under symmetric information would allow.

5 Conclusion

We have built a model of environmental tax on the assumption that a policymaker newly in office has better information regarding her environmental concern than would households. The case of enforceable environmental taxes is emphasized to answer the question of how much information environmental taxes alone can convey from the policymaker to households. This extends earlier analytical work on optimal environmental taxation by allowing for the possibility that environmental tax might be used by policymakers as a signal for her environmental concern.

In such a context, a green policymaker has an incentive to raise environmental tax above the Pigovian level that would prevail under symmetric information, in an attempt to trick households into believing that she has as little concern about the environment as a brown policymaker. Such a lie is expected to boost the private component of social welfare. The choice of the environmental tax may be distorted by the brown policymaker's need to transmit information on her true environmental concern.

Various results were found.

- Tax distortions are shown to emerge when the welfare obtained with the brown policymaker's Pigovian tax makes the green policymaker envious.

- Environmental taxes can fully reveal information on the policymaker's concern for the environment provided that the green policymaker, firstly, suffers a greater welfare loss from increasing tax, and secondly, generates a lower environmental damage with higher cleanup costs than does a brown policymaker.

- Selection of UE shows that households' prior beliefs of a brown policymaker must be sufficiently small for environmental taxes to fully reveal information. Otherwise, the selected UE are pooling, hence environmental taxes provide no information to households.

- The brown policymaker must distort her tax above the symmetric information Pigovian tax to be perfectly identified by households who unrealistically believe her to be green. By contrast, the brown policymaker finds it optimal to distort her tax below the symmetric information Pigovian tax and convey no information to households who believe more likely that she is brown. In such a pooling equilibrium, the green policymaker's choice of environmental tax is distorted upward.

It would be worthwhile to examine whether these predictions are robust to more general assumptions. In particular, the signaling framework could be extended into a dynamic setting similar to that in Noldeke and Van Damme (1990), which would allow markets to clear not only before but also after the environmental types of policymaker have sorted themselves. Another dynamic extension reminiscent of Bagwell and Riordan (1991) would be to consider that some economic agents are perfectly informed about the environmental concern of the policymaker and that their number grows as time passes. Then, it would become even more difficult for the green policymaker to trick households into believing that she is brown and the brown policymaker might be able to signal her true environmental concern with a smaller tax distortion. Finally, one intriguing extension would be to investigate a model in which other decision variables than the environmental tax might be simultaneously used as signals of environmental quality.

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